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On The Spin-Dependent Potential Between Heavy Quark And Antiquark

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Abstract

A new formula for the heavy quark-antiquark spin dependent potential is given by using the techniques developed in the heavy quark effective theory. The leading logarithmic quark mass terms emerging from the loop contributions are explicitly extracted and summed up. There is no renormalization scale ambiguity in this new formula. The spin-dependent potential in the new formula is expressed in terms of three independent color-electric and color-magnetic field correlation functions, and it includes both the Eichten-Feinberg's formula and the one-loop QCD result as special cases.

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The study of the structure of the spin-dependent (SD) interaction potential between heavy quark and antiquark from QCD is one of the interesting problems in heavy quark physics. So far there are mainly two kinds of approaches in the literature. The first kind of approach starts from the static limit (infinitely heavy quark limit) and makes relativistic corrections via the $1/m$ expansion, where m stands for the heavy-quark mass. The formula for the SD potential in this approach was first given by Eichten and Feinberg [1] in which the potential is expressed in terms of certain correlation functions of color-electric and color-magnetic fields weighted by the Wilson loop factor. Later Gromes [2] derived an important relation between the spin-independent (SI) and the SD potentials from the Lorentz invariance of the total potential and the correlation functions given in Ref. [1],and it supported the intuitive color-electric flux tube picture of color-electric confinement suggested by Buchmüller [3]. The second kind of approach is to calculate the SD potential from perturbative QCD up to one-loop level and put in the nonperturbative part of the potential by hand [4,5]. In this approach, certain terms containing $\ln m$ emerge from the loop contributions. Furthermore, in the case of unequal masses, new structure of the order-$1/m^2$ spin-orbit coupling containing $\ln m$ arises in this approach, which is not included in the first kind of approach [4,5]. It seems that there exists a discrepancy between these two kinds of approaches [6]. To understand the essence of this discrepancy and to have a deeper understanding of the $\ln m$-dependence in the SD potential has become an important theoretical problem in heavy quark physics for a quite long time. In a recent paper [7], two of us (Chen and Kuang) derived some general relations between the SI and SD potentials in the spirit of the $1/m$ expansion by using the technique of the reparameterization invariance [8–10] developed in the heavy quark effective theory (HQET), which include the Gromes relation and some new useful relations, and lead to the conclusion that the general structure of the SD potential is the same as that obtained in the second approach. However, such a simple symmetry argument does not concern the problem of the $\ln m$-dependence of the potential.

In this paper, we adopt the conventional formulation in the HQET to construct an effective QCD Lagrangian including both a heavy quark field and a heavy antiquark field,

with which we study the SD interaction potential. The conventional HQET has proved to be very powerful in studying the heavy-light quark systems [11,12,8–10,13,14] and most of the useful techniques developed in it can be applied to the present theory. We emphasize here that we are working at the quark level to study the interaction potential between heavy quarks rather than studying the bound state quarkonia. The reason causing the discrepancy between the two approaches can be easily understood in the effective Lagrangian formalism. When an effective Lagrangian is constructed by expanding the full theory in terms of $1/m$, the ultraviolet-behavior of the theory is changed. Therefore, when the order of the $1/m$ expansion and the loop integration are exchanged, which is involved in passing from the full theory to the effective theory, differences such as logarithmic quark mass terms can emerge. In order to reproduce the results of the full theory, one must match the full theory and the effective one. With this matching condition these logarithmic quark mass dependent terms can be explicitly extracted in the coefficients of higher dimensional operators and can be summed up by using the renormalization group equation (RGE). In the original work of Eichten and Feinberg [1] the authors expanded the full heavy fermion propagator in an "external" gluon field A^μ in terms of $1/m$. It is easy to see that each term in their expression corresponds to insertions of operators in the effective Lagrangian which contribute to the SD potential up to order $1/m^2$, but with only tree level coefficients. Therefore, the logarithmic terms are not accounted for. This is the essential reason that there is a discrepancy of the quark mass dependence between the Eichten-Feinberg-Gromes (EFG) formula and the one-loop calculation. With this insight, we can improve the EFG formula by summing the logarithmic quark mass terms and establish a consistent picture reconciling these two kinds of approaches in the framework of the effective Lagrangian. To this end, we need not only to renormalize the effective Lagrangian up to order of $1/m^2$, but also to consider the mixing of the nonlocal operators with local four fermion operators when we use the HQET to calculate the heavy quark-antiquark Green's functions. Then we can follow the methods developed in Ref. [15,1] and obtain a new formula for the spin dependent potential by using the renormalized effective Lagrangian. The above results of the two approaches are just two

special approximations of our new formula. Moreover, our new formula is independent of the renormalization scale parameter μ, so that it does not suffer from the scale ambiguity as the second approach does.

Let us first construct the renormalized effective Lagrangian for one heavy quark field up to order of $1/m^2$. We start from the full QCD Lagrangian. As in the conventional HQET, we define the heavy quark field $h_{v+}(x)$ and the heavy antiquark field $h'_{v-}(x)$ related to the original field $\psi(x)$ as [11]

$$h_{v+}(x) \equiv P_+ e^{imv\cdot x}\psi(x), \quad h'_{v-}(x) \equiv P_- e^{-imv\cdot x}\psi(x), \tag{1}$$

where v is the velocity of the heavy quark, and $P_\pm \equiv \dfrac{1 \pm \not{v}}{2}$. Integrating out the quantum fluctuation of the quark field, we obtain the heavy quark effective Lagrangian $\mathcal{L}_c$ [12,10]. After expanding order by order in powers of $1/m$, the first three terms are

$$\mathcal{L}_0 = c_0 \bar{h}_{v+}(x) iD\cdot v h_{v+}(x), \tag{2}$$

$$\mathcal{L}_1 = c_0 c_1 \bar{h}_{v+}(x)\frac{(iD)^2}{2m}h_{v+}(x) - c_0 c_2 \bar{h}_{v+}(x)\frac{(iD\cdot v)^2}{2m}h_{v+}(x) + c_0 c_3 g_s \bar{h}_{v+}(x)\frac{G_{\mu\nu}\sigma^{\mu\nu}}{4m}h_{v+}(x), \tag{3}$$

$$\begin{aligned}\mathcal{L}_2 = {} & c_0 \frac{g_s}{4m^2}\bar{h}_{v+}(x)(c_4 v^\nu D^\mu G_{\mu\nu} + ic_5 \sigma^{\mu\nu} v^\sigma D_\mu G_{\nu\sigma})h_{v+}(x) \\ & -\frac{c_0}{4m^2}\bar{h}_{v+}(x)\left[c_6 (iD)^2 - ic_7 (D\cdot v)^2 - c_8 \frac{g_s}{2} G_{\mu\nu}\sigma^{\mu\nu}\right] iD\cdot v h_{v+}(x),\end{aligned} \tag{4}$$

where $iD_\mu = i\partial_\mu - gA^a_\mu T^a$. The last term in $\mathcal{L}_2$ has no contribution in order $1/m^2$ due to the equation of motion. At tree level the c_i's are all unity. Note that after the expansion the high energy behavior is different from that in the full theory. So the operators in (2)-(4) need to be renormalized and their coefficients can be determined by matching to the full theory. Here $\sqrt{c_0}$ corresponds to the wavefunction renormalization constant, and $c_1 - c_3$ have been calculated in Refs. [13] and [14], using the RG summation, Ref. [14] gives

$$c_1(\mu, m) = 1, \quad c_2(\mu, m) = 3\left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{-\frac{8}{25}} - 2, \quad c_3(\mu, m) = \left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{-\frac{9}{25}}, \tag{5}$$

where $c_i \equiv c_i(\mu, m)$. The coefficients $c_4 - c_6$ can be determined by the reparameterization invariance of $\mathcal{L}_c$ which leads to the reparameterization invariance of the renormalized

Lagrangian. This is shown as follows: Consider the infinitesimal velocity transformation $v \to v + \Delta v$ [10]. The infinitesimal transformation of $\delta\mathcal{L}$ can be written as

$$\delta\mathcal{L} = \delta\mathcal{T}_0 + \frac{1}{2m}\delta\mathcal{T}_1, \tag{6}$$

where

$$\delta\mathcal{T}_0 = c_0(1-c_1)\bar{h}_{v+}(x) iD\cdot\Delta v h_{v+}(x), \tag{7}$$

and

$$\begin{aligned}\delta\mathcal{T}_1 \;=\;& c_0(1-2c_2+c_6)\;\bar{h}_{v+}(x) iD\cdot\Delta v iD\cdot v h_{v+}(x)\\ &+\; c_0(c_3-c_2+\tfrac{1}{2}c_4-\tfrac{1}{2}c_5)\;\bar{h}_{v+}(x)\gamma^\mu v^\nu igG_{\mu\nu}\frac{\Delta\!\!\!/ v}{2}h_{v+}(x)\\ &+\; c_0(1-c_2-c_3+\tfrac{1}{2}c_4+\tfrac{1}{2}c_5)\;\bar{h}_{v+}(x)\frac{\Delta\!\!\!/ v}{2}\gamma^\mu v^\nu igG_{\mu\nu}h_{v+}(x).\end{aligned} \tag{8}$$

Different terms in (7)-(8) are of different Lorentz structures, and therefore, they should vanish separately. $\delta\mathcal{T}_0 = 0$ leads to $c_1 = 1$, i.e., the kinetic energy term is not renormalized [9], and $\delta\mathcal{T}_1 = 0$ gives the two relations

$$c_4 = c_6 = 2c_2 - 1, \qquad c_5 = 2c_3 - 1. \tag{9}$$

Using Eq.(5) we obtain

$$c_4(\mu,m) = 6\left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{-\frac{8}{25}} - 5, \qquad c_5(\mu,m) = 2\left(\frac{\alpha_s(\mu)}{\alpha_s(m)}\right)^{-\frac{9}{25}} - 1. \tag{10}$$

The effective Lagrangian for antiquark field can be obtained by simply replacing v by $-v$ and $h_{+v}(x)$ by $h'_{-v}(x)$ in the above effective Lagrangian.

To study the heavy quark-antiquark interaction potential we are going to evaluate the heavy quark-antiquark four point Green's function to order $1/m^2$. In doing this, we need to calculate all the possible order $1/m$ operator insertions. Then, additional divergences will appear from double insertions of these operators, i.e., these bilocal operators will mix with certain local four-fermion operators. Let us consider a general unequal mass case, e.g. the heavy quark Q_1 with mass m_1 and the antiquark $\bar{Q}_2$ with mass m_2, as in the case of the

$c\bar{b}$. To order $1/m^2$, there are only two local dimension 6 color singlet four fermion operators $O_1(x)$ and $O_2(x)$:

$$O_1(x) = \frac{g_s^2}{4m_1 m_2}\bar{h}_{v+i}(x)\sigma^{\mu\nu}h_{v+j}(x)\bar{h}'_{v-j}(x)\sigma_{\mu\nu}h'_{v-i}(x), \tag{11}$$

$$O_2(x) = \frac{g_s^2}{4m_1 m_2}\bar{h}_{v+i}(x)\sigma^{\mu\nu}h_{v+i}(x)\bar{h}'_{v-j}(x)\sigma_{\mu\nu}h'_{v-j}(x), \tag{12}$$

where $i, j = 1, 2, \cdots, N_c$ are color indices. Suppose $m_1 > m_2$. The heavy quark antiquark effective Lagrangian up to order $1/m_1^2$, $1/m_1 m_2$, and $1/m_2^2$ can be constructed in two steps as follows. Starting from the Lagrangian in the full theory, we first treat Q_1 as a heavy quark, and obtain

$$\mathcal{L}' = \mathcal{L}_{Q_1 eff} + \mathcal{L}_{Q_2}. \tag{13}$$

We do not need to add new operators in (13) because there are no divergent terms of the form of $O_1(x)$ and $O_2(x)$. Next, we treat Q_2 as a heavy antiquark, and obtain

$$\mathcal{L}'' = \mathcal{L}_{Q_1 eff} + \mathcal{L}_{Q_2 eff} + d_1(\mu)O_1(\mu) + d_2(\mu)O_2(\mu), \tag{14}$$

where the last two terms are the two necessary dimension-6 operators with unknown coefficients $d_1(\mu)$ and $d_2(\mu)$, respectively. Now we determine $d_1(\mu)$ and $d_2(\mu)$ by using the RGE. It is easy to see that only the magnetic operator insertion in each fermion line will mix with O_1 and O_2 due to the Lorentz structure. Let us denote the resulting contribution by

$$O_0(x) \equiv \frac{g_s^2}{16 m_1 m_2}\int d^4y T^* \left[\bar{h}_{v+}(x)G_{\mu\nu}\sigma^{\mu\nu}h_{v+}(x)\bar{h}'_{v-}(y)G_{\alpha\beta}\sigma^{\alpha\beta}h'_{v-}(y)\right], \tag{15}$$

and its coefficient by $d_0(\mu)$. Here T^* means time ordering. The coefficients $d(\mu) \equiv (d_0(\mu),\ d_1(\mu),\ d_2(\mu))$ satisfy the renormalization group equation

$$\mu\frac{d}{d\mu}d(\mu) + d(\mu)\gamma = 0, \tag{16}$$

where γ is the anomalous dimension matrix. A straightforward one-loop calculation using the HQET technique gives

$$\gamma = \frac{g^2}{4\pi^2} \begin{pmatrix} -N_c & \frac{N_c}{8} & -\frac{1}{8} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{17}$$

Here $\gamma_{00} = 2\gamma_{mag}$. The results $\gamma_{10} = \gamma_{20} = 0$ means that local operators are not able to mix with bilocal operators. However, bilocal operators can mix with local operators, and there are two box, two cross, and a "fish" diagram contributing to γ_{01} and γ_{02}. That $\gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 0$ can be understood as follows: The anomalous dimensions are gauge independent. If we take an axial gauge where $v \cdot A = 0$ gluonic interactions decouple from the fermion field in the zeroth order effective Lagrangian (2). The initial condition for the RGE is determined by matching the effective theory to the full theory at $\mu = m_2$, i.e.

$$d(m_2) = (\ c_3(m_2, m_1), 0, 0\ ) . \tag{18}$$

With this, the solution of the RGE (16) are

$$\begin{aligned} d_0(\mu) &= c_3(\mu, m_2)c_3(\mu, m_1) = \left(\frac{\alpha_s^2(\mu)}{\alpha_s(m_1)\alpha_s(m_2)}\right)^{-\frac{9}{25}} , \\ d_1(\mu) &= \frac{1}{8}c_3(m_2, m_1)[1 - c_3^2(\mu, m_2)] = \frac{1}{8}\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)^{-\frac{9}{25}} \left[1 - \left(\frac{\alpha_s(\mu)}{\alpha_s(m_2)}\right)^{-\frac{18}{25}}\right] , \\ d_2(\mu) &= -\frac{1}{N_c}d_1(\mu). \end{aligned} \tag{19}$$

Our effective Lagrangian (14) is thus completely determined.

Now we apply it to calculate spin-dependent force. We shall take $v = (1, 0, 0, 0)$ and denote $h_{v+}(x)$, $h'_{v-}(x)$ as $h(x)$, $h'(x)$ for short. Similar to Ref. [1], we introduce a gauge invariant four-point Green's function

$$I = \langle 0|T^*[\bar{h}'(y_2)\bar{\Gamma}_B P(y_2, y_1)h(y_1)]\,[\bar{h}(x_1)\Gamma_A P(x_1, x_2)h'(x_2)]\,|0\rangle, \tag{20}$$

where $P(x, y) \equiv P \exp\left[ig \int_y^x dz_\mu A^\mu(z)\right]$ is the path-ordered exponential [16,17]. As is argued in Refs. [15,1], in the limit that the time interval $T \equiv (y_1^0 + y_2^0)/2 - (x_1^0 + x_2^0)/2 \to \infty$, with $x_2^0 - x_1^0$ and $y_2^0 - y_1^0$ fixed, the limit of I is

$$I \to \delta_{AB}\delta(\vec{r}_x - \vec{r}_y) \exp[-T\epsilon(r)], \tag{21}$$

where $\vec{r}_x = \vec{x}_1 - \vec{x}_2$, $\vec{r}_y = \vec{y}_1 - \vec{y}_2$, $r = |\vec{r}_x|$, and $\epsilon(r)$ is just the static energy between the quark and antiquark separated by the spatial distance r. Here the appropriate ordering of the limits is that first $m \to \infty$ and then $T \to \infty$, so that the motion of the quark and the antiquark can be treated perturbatively [15,1].

Taking all the operators with dimension higher than 4 in the Lagrangian as the perturbative part, I can be calculated by using standard perturbation theory. In the calculation, the zeroth order full fermion propagator $S_0(x, y, A)$ in the external gluon field A^μ is used and it is [15,1]

$$S_0(x, y, A) = -i\theta\ (x_0 - y_0)P(x_0, y_0)\delta(\vec{x} - \vec{y}). \tag{22}$$

Next we define the symbol $\langle \cdots \rangle$ and $\tilde{I}$ as

$$\langle Q(x)\rangle \equiv \int [dA^\mu] Tr \left\{ P\left[\exp\left(ig \oint_{C(r,T)} dz_\mu A^\mu(z) \right) Q(x) \right] \right\}_{x\in C} \exp(iS_{YM}(A)), \tag{23}$$

$$I \equiv Tr(P_+ \bar{\Gamma} P_- \Gamma \tilde{I})\delta(\vec{x}_1 - \vec{y}_1)\delta(\vec{x}_2 - \vec{y}_2). \tag{24}$$

To order $1/m_1^2$, $1/(m_1 m_2)$ and $1/m_1^2$, $\tilde{I}$ can be expressed as

$$\begin{aligned} \tilde{I} \;=\; & \langle 1\rangle + i\left[\int_{T/2}^{T/2} dz \frac{1}{m_1} \langle \mathbf{D}^2(\mathbf{x}_1, z) - c_3(\mu, m_1) g_s(\mu) \vec{\sigma}_1 \cdot B(\mathbf{x}_1, z)\rangle + (1 \leftrightarrow 2) \right] \\ - & \frac{g_s(\mu)}{4m_1^2} \int_{T/2}^{T/2} dz [(c_4(\mu, m_1)\delta_{ij} - c_5(\mu, m_1) i\epsilon_{ijk}\sigma_1^k)\langle E^i(\mathbf{x}_1, z) D^j(\mathbf{x}_1, z)\rangle + (1 \leftrightarrow 2) \\ - & \frac{1}{m_1^2} \int_{-T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \theta(z' - z)[\langle(\mathbf{D}^2 - c_3(\mu, m_1) g_s(\mu)\sigma_1^i B^i)(\mathbf{x}_1, z) \\ & (\mathbf{D}^2 - c_3(\mu, m_1) g_s(\mu)\sigma_1^i B^i)(\mathbf{x}_1, z')\rangle + (1 \leftrightarrow 2)] - \frac{1}{m_1 m_2} \int_{T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \\ & \langle(\mathbf{D}^2 - c_3(\mu, m_1) g_s(\mu)\sigma_1^i B^i)(\mathbf{x}_1, z)(\mathbf{D}^2 - g_s(\mu) c_3(\mu, m_2)\sigma_2^j B^j)(\mathbf{x}_2, z')\rangle \\ + & \frac{N_c g_s^2(\mu)}{2m_1 m_2} T d(\mu)\sigma_1^i \sigma_2^i \delta^3(\mathbf{x}_1 - \mathbf{x}_2), \end{aligned} \tag{25}$$

where

$$d(\mu) = d_1(\mu) + \frac{d_2(\mu)}{N_c}. \tag{26}$$

Similar to the derivation given in Ref. [1], we obtain the spin-dependent potential

$$
\begin{aligned}
V(r) \;=\; & V_0(r) + \left(\frac{\mathbf{S_1}}{m_1^2} + \frac{\mathbf{S_2}}{m_2^2}\right) \cdot \mathbf{L} \left[\left(c_+(\mu, m_1, m_2) - \frac{1}{2} \right) \frac{dV_0(r)}{dr} \right. \\
& + \left. c_+(\mu, m_1, m_2) \frac{dV_1(\mu, r)}{dr} \right] + \left(\frac{\mathbf{S_1} + \mathbf{S_2}}{m_1 m_2} \right) \cdot \mathbf{L} c_+(\mu, m_1, m_2) \frac{1}{r} \frac{dV_2(\mu, r)}{dr} \\
& + \frac{1}{m_1 m_2} \frac{(\mathbf{S_1} \cdot \mathbf{r})(\mathbf{S_2} \cdot \mathbf{r}) - \frac{1}{3} \mathbf{S_1} \cdot \mathbf{S_2}\, r^2}{r^2} c_3(\mu, m_1) c_3(\mu, m_2) V_3(\mu, r) \\
& + \frac{1}{3} \frac{1}{m_1 m_2} \mathbf{S_1} \cdot \mathbf{S_2} \left[(c_3(\mu, m_1) c_3(\mu, m_2) V_4(\mu, r) - 6 N_c g_s^2(\mu) d(\mu) \delta(\mathbf{r}) \right] \\
& + \left(\frac{\mathbf{S_1}}{m_1^2} - \frac{\mathbf{S_2}}{m_2^2} \right) \cdot \mathbf{L} c_-(\mu, m_1, m_2) \frac{1}{r} \frac{d[V_0(\mu, r) + V_1(\mu, r)]}{dr} \\
& + \left(\frac{\mathbf{S_1} - \mathbf{S_2}}{m_1 m_2} \right) \cdot \mathbf{L} c_-(\mu, m_1, m_2) \frac{1}{r} \frac{dV_2(\mu, r)}{dr},
\end{aligned}
\tag{27}
$$

where

$$
c_+(\mu, m_1, m_2) = \frac{1}{2}[c_3(\mu, m_1) + c_3(\mu, m_2)], \quad c_-(\mu, m_1, m_2) = \frac{1}{2}[c_3(\mu, m_1) - c_3(\mu, m_2)], \tag{28}
$$

and

$$
V_0(r) \equiv -\lim_{T\to\infty} \frac{\ln\langle 1\rangle}{T}, \tag{29}
$$

$$
r_k \frac{1}{r} \frac{dV_1(\mu, r)}{dr} \equiv \lim_{T\to\infty} \epsilon_{ijk} \int_{-T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \left(\frac{z' - z}{T} \right) g_s^2(\mu)/2 \langle B^i(\mathbf{x}_1, z) E^j(\mathbf{x}_1, z') \rangle / \langle 1 \rangle, \tag{30}
$$

$$
r_k \frac{1}{r} \frac{dV_2(\mu, r)}{dr} \equiv \lim_{T\to\infty} \epsilon_{ijk} \int_{-T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \left(\frac{z'}{T} \right) g_s^2(\mu)/2 \langle B^i(\mathbf{x}_2, z) E^j(\mathbf{x}_1, z') \rangle / \langle 1 \rangle, \tag{31}
$$

$$
[(\hat{r}_i \hat{r}_j - \frac{\delta^{ij}}{3}) V_3(\mu, r) + \frac{\delta^{ij}}{3} V_4(\mu, r)] \equiv \lim_{T\to\infty} \int_{-T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \frac{g_s^2(\mu)}{T} \langle B^i(\mathbf{x}_1, z) B^j(\mathbf{x}_2, z') \rangle / \langle 1 \rangle. \tag{32}
$$

In Ref. [2], Gromes derived a relation $\frac{d}{dr}[V_0(r) + V_1(\mu, r) - V_2(\mu, r)] = 0$. The relation $c_5(\mu, m) = 2c_3(\mu, m) - 1$ in (9) obtained from reparameterization invariance ensures that the general relations derived in Ref. [7] are all satisfied. It also shows that those relations are consistent with each other. Using those relation, the SD potential can be simplified as

$$
\begin{aligned}
V(r) &= V_0(r) + \left(\frac{\mathbf{S_1}}{m_1^2} + \frac{\mathbf{S_2}}{m_2^2}\right) \cdot \mathbf{L} \left(c_+(\mu, m_1, m_2)\frac{dV_2(\mu, r)}{dr} - \frac{1}{2}\frac{dV_0(r)}{dr}\right) \\
&+ \left(\frac{\mathbf{S_1} + \mathbf{S_2}}{m_1 m_2}\right) \cdot \mathbf{L} c_+(\mu, m_1, m_2)\frac{1}{r}\frac{dV_2(\mu, r)}{dr} \\
&+ \frac{1}{m_1 m_2}\frac{(\mathbf{S_1}\cdot\mathbf{r})(\mathbf{S_2}\cdot\mathbf{r}) - \frac{1}{3}\mathbf{S_1}\cdot\mathbf{S_2}\, r^2}{r^2} c_3(\mu, m_1)c_3(\mu, m_2)V_3(\mu, r) \\
&+ \frac{1}{3}\frac{1}{m_1 m_2}\mathbf{S_1}\cdot\mathbf{S_2}\left[(c_3(\mu, m_1)c_3(\mu, m_2)V_4(\mu, r) - 6N_c g_s^2(\mu) d(\mu)\delta(\mathbf{r})\right] \\
&+ \left[\left(\frac{\mathbf{S_1}}{m_1^2} - \frac{\mathbf{S_2}}{m_2^2}\right) \cdot \mathbf{L} + \left(\frac{\mathbf{S_1} - \mathbf{S_2}}{m_1 m_2}\right) \cdot \mathbf{L}\right] c_-(\mu, m_1, m_2)\frac{1}{r}\frac{dV_2(\mu, r)}{dr}.
\end{aligned}
\tag{33}
$$

This is our new formula for the spin-dependent quark-antiquark potential.

In Eq. (33), if we take each coefficient to be its tree level value, i.e., $c_3(\mu, m) = c_+(\mu, m_1, m_2) = 1$ and $c_-(\mu, m_1, m_2) = d(\mu) = 0$, our result reduces to the EFG formula. Next we compare our result with that in the one-loop QCD calculation. First we see that $V_5(\mu, m_1, m_2)$ introduced in Ref. [5] is not an independent function. If we take the one-loop values of $c_3(\mu, m)$, $c_\pm(\mu, m_1, m_2)$, $d(\mu)$, and then calculate the correlation functions to one-loop, our formula (33) reproduces all the logarithmic mass terms in Refs. [4,5].

In conclusion, the leading logarithmic quark mass terms emerging from the loop contributions are explicitly extracted and summed up by matching the effective theory and the full theory and solving the renormalization group equation. The discrepancy appearing in the EFG results and one-loop calculation can then be understood. Our result shows that the effective theory can reproduce the full theory beyond tree level in $1/m^2$ and can be used in calculating the Green's functions with two heavy quark external lines.

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